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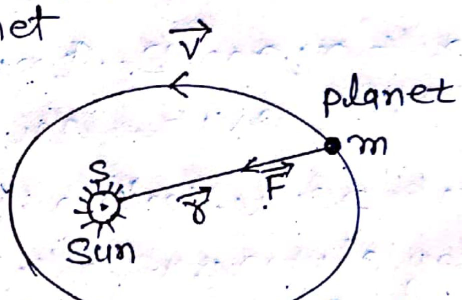
Keplar's Law of planetary motion:-

Our Solar System consists of the Sun and the planets which are revolving round the Sun in different orbits. Keplar studied this planetary motion and gave the following three laws

1. <u>First Law</u>:- Every planet moves around the Sun in an elliptical orbit with Sun as one focus. This implies that the orbit lies in a plane.

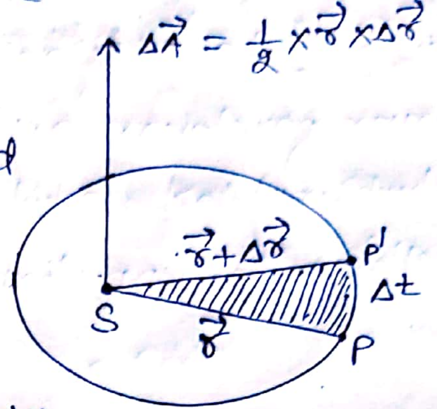
<u>Proof</u>:- Let the planet moves in elliptical orbits around the Sun with one focus of the ellipse at the Centre of the Sun S. The gravitational force $\vec{F}$ exerted by the Sun or the planet is always directed towards S. Thus the planet moves under a central force. Then the angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ of the planet with respect to S is Constant in magnitude and direction.



Since both r and v are always perpendicular to $\vec{L}$, then the orbit of always lies in a plane perpendicular to $\vec{L}$. This is Keplar's First Law.

2. <u>Second Law:</u>– The line joining the Sun and the planet sweeps equal area in equal intervals of time.

<u>Proof:</u>– Let S be the centre of the Sun and P that of the planet, of mass m in its orbit. Let $\vec{r}$ be the radius vector of the planet with respect to S. Suppose that in a small time interval Δt, the planet moves from P to P' where the radius vector is $\vec{r}+\Delta\vec{r}$.



Then the vector area $\Delta\vec{A}$ swept by the radius vector in time-interval Δt is

$$\Delta\vec{A} = \frac{1}{2} \times \vec{r} \times \Delta\vec{r}$$

$$\frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2} \times \vec{r} \times \frac{\Delta\vec{r}}{\Delta t}$$

$$\therefore \lim_{\Delta t \to 0} \frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2} \times \vec{r} \times \left(\lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} \right)$$

$$\text{or, } \frac{d\vec{A}}{dt} = \frac{1}{2} \times \vec{r} \times \left(\frac{dr}{dt} \right)$$

$$\text{or, } \frac{d\vec{A}}{dt} = \frac{1}{2} \times \vec{r} \times \vec{v} \quad \left(\because \vec{v} = \frac{d\vec{r}}{dt} \right)$$

$$\text{or, } \frac{d\vec{A}}{dt} = \frac{1}{2m} \times \vec{r} \times m\vec{v}$$

$$\because \vec{r} \times m\vec{v} = L \text{ (Angular momentum)}$$

$$\therefore \frac{d\vec{A}}{dt} = \frac{1}{2m} \cdot \vec{L}$$

$\because$ L is a Constant for central force.

$$\therefore \frac{d\vec{A}}{dt} = \text{Constant} \quad (\text{Areal velocity})$$

Thus radius vector of the planet sweeps out equal area in equal interval of times.

This is Keplar's Second Law

3. Third Law :- The square of the period of any planet about the Sun is proportional to the Cube of the Semi-major axis of the elliptic orbit.

proof :- Let T be the period of revolution in an orbit, and ~~a are the~~ a and b are the Semi-major and Semi-minor axis of the ellipse.

Then area of the ellipse $= \pi ab$

The areal velocity is

$$\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}$$

$$\because T = \frac{\text{Area of ellipse}}{\text{Areal Velocity}}$$

$$= \frac{\pi ab}{dA/dt}$$

$$= \frac{\pi ab}{L/2m}$$

$$\therefore T = \frac{2\pi mab}{L}$$

$$\therefore T^2 = \frac{4\pi^2 m^2 a^2 b^2}{L^2}$$

If l be the latus rectum of the ellipse then

$$l = \frac{b^2}{a}$$

$$\therefore b^2 = al$$

$$\therefore T^2 = \frac{4\pi^2 m^2 a^2 . al}{L^2}$$

$$T^2 = \frac{4\pi^2 m^2 a^3 . l}{L^2}$$

$$\therefore T^2 = \left(\frac{4\pi^2 m^2 l}{L^2}\right) a^3$$

$$\therefore T^2 = Ka^3 \quad \left(K = \frac{4\pi^2 m^2 l}{L^2} = \text{Constant} \right)$$

$$\therefore T^2 \propto a^3$$

This is keplar's third Law.